Canonique : N fixé

$Z = \int d\Gamma \, e^{-\beta H}$

$E = -\frac{\partial \ln Z}{\partial \beta}$; $F = -\frac{\ln Z}{\beta}$

$\mu = \frac{\partial F}{\partial N}\Big|_{T,V}$

$P = \frac{1}{\beta} \frac{\partial \ln Z}{\partial V}\Big|_{\beta,N}$

Grand Canonique : N varie

$Q = \sum_{N=0}^{\infty} z^N Z_N$ et $Z_N = \int d\Gamma \, e^{-\beta H}$

avec la fugacité $z = e^{\beta\mu}$

$E = -\frac{\partial \ln Q}{\partial \beta}\Big|_{z,V}$

$\overline{N} = z \frac{\partial \ln Q}{\partial z}\Big|_{\beta,V}$

$P = \frac{1}{\beta} \frac{\partial \ln Q}{\partial V}\Big|_{z,\beta}$

- Si sans intéractions $\exists \, \zeta$
  sinon (= intéraction) on peut
  sur $Z = \sum_m \Omega_m \, e^{-\beta E_m}$
- Si indiscernables $Z = \zeta^N$
  Si discernables $Z = \zeta^N C_m^N$

Probabilité :

$P_C = \frac{e^{-\beta H}}{Z}$ || $P_{GC} = \frac{e^{-\beta H + \beta\mu}}{Q}$

Théorème équipartition :

chaque degré de liberté (ν) quadratique contribue pour $k\frac{T}{2}$ à $\langle H \rangle$:

$\langle H \rangle = \nu \frac{kT}{2}$

Fermions : spin demi-entier

principe d'exclusion de Pauli

Fonction d'onde antisymétrique

$$Q_{FD} = \prod_{\ell} \left[1 + z\, e^{-\beta \varepsilon_\ell} \right]$$

$$\langle n_\ell \rangle_{FD} = \frac{1}{e^{\beta(\varepsilon_\ell - \mu)} + 1}$$

$\varepsilon_F = kT_F$ Ex : $e^-, n, p, {}^3He$

Bosons : spin entier

Fonction d'onde symétrique

$$Q_{BE} = \prod_{\ell} \frac{1}{1 - z e^{-\beta \varepsilon_\ell}}$$

$$\langle n_\ell \rangle_{BE} = \frac{1}{e^{\beta(\varepsilon_\ell - \mu)} - 1}$$

Ex : photons ; ${}^4He$, phonons

- Distribution de Fermi :

$$f(x) = \frac{1}{e^{\beta(\varepsilon_\ell - \mu)} + 1}$$

à $T=0$ $f(\varepsilon) = \Theta(\varepsilon_F - \varepsilon)$

- Si pas de loi de conservation alors $\mu = 0$
- $\mu = \varepsilon_F$ à $T=0$

$d=1$ $\rho(P) = \frac{2L}{h}$

$d=2$ $\rho(P) = \frac{2\pi P S}{h^2}$

$d=3$ $\rho(P) = \frac{4\pi P^2 V}{h^3}$

$$N = \int_0^\infty f(\varepsilon)\, \rho(\varepsilon)\, d\varepsilon$$

$$E = \int_0^\infty \varepsilon\, f(\varepsilon)\, \rho(\varepsilon)\, d\varepsilon$$

En général :

$$Q = \underbrace{\sum_{\{n_\ell\}} \prod_{\ell}}_{= \prod_\ell \sum_{n_\ell}} \left[z\, e^{-\beta \varepsilon_\ell} \right]^{n_\ell}$$

$$\langle n_\ell \rangle = \frac{1}{Q} \prod_{\ell} \sum_{n_\ell} n_\ell z^{n_\ell} e^{-\beta \varepsilon_\ell n_\ell} = -\frac{1}{\beta} \frac{\partial \ln Q}{\partial \varepsilon_\ell}$$

$z \ll 1$ approximation classique

Antisymétrie :

$$f_A(x) = \frac{1}{2}\left[f(x) - f(-x) \right]$$

Symétrie :

$$f_S(x) = \frac{1}{2}\left[f(x) + f(-x) \right]$$

Au sens des distributions :

$$\langle \delta', \varphi \rangle = -\langle \varphi', \delta \rangle$$

Wien displacement law :

$$\frac{\omega_m^1}{T_1} = \frac{\omega_m^2}{T_2}$$

Debye approximation :

$\omega_k = c_s k \quad \forall k$

Bose Einstein Condensation:

when the # of particles in the ground state is macroscopic

$$\lim_{\substack{N\to\infty \\ \frac{N}{V}=cste}} \frac{\langle n_0 \rangle}{N} \text{ reste finie}$$

Longueur d'onde thermique

$$\lambda = \frac{h}{(2\pi m k T)^{1/2}}$$

Sommerfeld:

$$f(\varepsilon) = \Theta(\mu-\varepsilon) - \frac{\pi^2}{6}(kT)^2\, \delta'(\varepsilon-\mu)$$

Gaz parfait:

$$\mu = -kT \ln\left(\frac{kT}{P}\,\frac{1}{\lambda^3}\right)$$

Ideal gas:

$$Z_K = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N$$

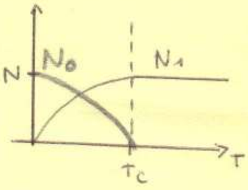
Typical length between particles:

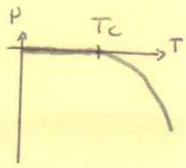
$$d \sim \left(\frac{V}{N}\right)^{1/3}$$

Total # of particles:

$$N = (2S+1)\int_0^{p_F} d^3p\, \rho(p)$$

with $d^3p = p^2\,dp\,\underbrace{d\Omega}_{=4\pi}$ and $\rho(p) = \left(\frac{L}{h}\right)^D$





$\underline{T<T_c}$: $N = N_0 + N_1$

$$\text{avec } N_0 = \frac{1}{z^{-1}e^{\beta\varepsilon_0} \pm 1} = \frac{z}{e^{\beta\varepsilon_0} \pm z}$$

$$N_1 = \int_{\varepsilon_0}^{\infty} \frac{\rho(\varepsilon)\,d\varepsilon}{e^{\beta(\varepsilon-\mu)} \pm 1}$$

$\underline{T=T_c}$: $N = N_1$

$$\text{avec } N_1 = \int_{\varepsilon_0}^{\infty} \frac{\rho(\varepsilon)\,d\varepsilon}{e^{\beta_c(\varepsilon-\varepsilon_0)} \pm 1}$$

$$\text{et } \mu = \varepsilon_0$$

On peut rescaller l'énergie pour avoir $\varepsilon_0 = 0$

Black body:

$$P = \frac{1}{3}\sigma' T^4 \; ; \; \frac{E}{V} = \sigma' T^4$$

$$\phi = \sigma T^4 \qquad \sigma, \sigma' = cste$$

$\uparrow$ flux

$$\int_0^\infty dx\, \frac{x^n}{e^x - 1} = \Gamma(n+1)\,\zeta(n+1)$$

$$\int_0^\infty dx\, \frac{x^n}{e^x + 1} = \left(1 - \frac{1}{2^n}\right)\Gamma(n+1)\,\zeta(n+1)$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt$$